Appendix A. Adjusting the retirement cap authority and remittance requirements

The Commission's retirement rate authority (RRA) is flawed in that the additional cap space r_t from Formula IV-1 does not generate sufficient revenue to cover the remittance required under Formula IV-3. Correcting the flaw requires resolution of two issues. First, the formulas need to recognize that a price increase of r_t percent results in some volume loss, unless demands are perfectly inelastic. The effects of the volume loss partly offset the increased revenue due to increased prices. *See, e.g.*, Supp. Decl. of Timothy J. Brennan for the Public Representative, PRC Docket No. RM2017-3 (Mar. 1, 2018), at 4-5. Second, the formulas need to account for the fact that the volume loss resulting from the RRA will induce some additional rate authority (compared to a baseline with no RRA) as the lost volumes will feed into the density-based rate authority with a lag. The second factor partly offsets the first, so accounting for both is necessary to balance the revenue from the RRA with the required remittance.

There are two options for balancing the RRA with the required remittance, while remaining within the general framework of the Commission's proposal. In Option 1, the rate authority may be adjusted upward so that the cap space produces a revenue yield of r_t on an "after-rates" basis. In Option 2, the rate authority r_t from Formula IV-1 is retained, but the revenue accumulation term is adjusted downward to account for the net volume loss and density effects.

I. Option 1: Add a formula for retirement rate authority (r^*t) needed to fund the Commission's proposed remittance obligation on an after-rates basis

To implement Option 1, we determine the amount of cap space r^*_t required so that the after-rates net revenue will yield the required remittance from accumulating r_t in Formula IV-3. The actual required cap space is an amount $r^*_t > r_t$ that satisfies the equation:

$$\left(1 - \frac{1}{1 + r_t}\right) MDR(r_t^*)_{t,AR} = MDR(r_t^*)_{t,AR} - MDR_{t,BR} . \tag{1}$$

The left-hand side of the equation (1) may be interpreted as the portion of the accumulated remittance requirement associated with the use of the cap space r^*_{t} , and the right-hand side is the after rates revenue generated by r^*_{t} . Simplifying and rearranging terms, we obtain

$$\left(1 - \frac{r_t}{1 + r_t}\right) MDR(r_t^*)_{t,AR} = MDR_{t,BR} , \qquad (1')$$

where $MDR_{t,AR}$ and $MDR_{t,BR}$ are market dominant revenue in period t after and before the application of the RRA (r^*_t) . $MDR_{t,AR}$ is a function of r^*_t .

Let x_t denote other rate authority in period t excluding the RRA, d_t denote rate authority induced by the RRA, t be the change in the CPI price index, and t be the aggregate own-price demand elasticity applicable to market dominant products. The aggregate demand elasticity is measured as a volume-weighted average of product-level elasticities. Then unit revenue t and market dominant volume t, with the application of period t RRA (t), will be

$$P_{t,AR} = (1 + x_t + d_t + r_t^*)P_{t-1}$$
 and

$$V_{t,AR} = \left(\frac{1 + x_t + d_t + r_t^*}{(1 + c_t)}\right)^{\varepsilon} V_{t-1} = (1 + x_t + d_t + r_t^*)^{\varepsilon} (1 + c_t)^{-\varepsilon} V_{t-1} ,$$

where

$$\varepsilon = \frac{1}{V_t} \sum_{i=1}^N V_{i,t} \varepsilon_i \ .$$

In the volume equation, the nominal change in the price level $1 + x_t + d_t + r_t^*$ is deflated by CPI with $1 + c_t$. Only the rate increase in excess of CPI raises real prices and thus is relevant for determining the price-driven change in the quantity demanded.

In the absence of the RRA, the change in market dominant prices and volumes depend only on the other rate authority X_t :

$$P_{t,BR} = (1 + x_t)P_{t-1}$$

$$V_{t,BR} = \left(\frac{1+x_t}{1+c_t}\right)^{\varepsilon} V_{t-1}$$

$$MDR_{t-1} = P_{t-1}V_{t-1}$$
.

As noted above, a revised formula must also account for the fact that RRA-induced volume loss will result in more density rate authority (all other things being equal). The induced effect of the RRA on the density authority d_t may be written as

$$d_t = -(IC/TC)_t * (((1 + x_{t-1} + d_{t-1} + r_{t-1}^*)/(1 + x_{t-1}))^{\varepsilon} - 1), \qquad (2)$$

where IC/TC is the institutional cost share, and $d_{t-1} = 0$ in the first year where retirement rate authority applies. Since the retirement rate authority does not affect delivery points or competitive product volumes, the induced change in density is due to the lagged effect on market-dominant volume. This is not an explicit component of the RRA, but rather arises due to interaction between the RRA and the density-based authority.

¹ Under the Commission's proposal, the RRA may induce rate authority through the density-based cap authority via lagged volume losses.

Compensating for this interaction in the rate authority or remittance amount (as appropriate) avoids "double-counting" of a portion of the revenue from both forms of rate authority.

We may thus rewrite equation (1') as:

$$\left(1 - \frac{r_t}{1 + r_t}\right) (1 + x_t + d_t + r_t^*)^{\varepsilon + 1} (1 + c_t)^{-\varepsilon} MDR_{t-1} = (1 + x_t)^{\varepsilon + 1} (1 + c_t)^{-\varepsilon} MDR_{t-1} . \tag{3}$$

Canceling and rearranging terms,

$$(1 + x_t + d_t + r_t^*)^{\varepsilon+1} = (1 + r_t)(1 + x_t)^{\varepsilon+1}$$
. (4)

Solving for r_t^* for equation (4), we obtain the needed rate authority in period t as a function of r_t to generate sufficient revenue to fund the remittances under Formula IV-3:

$$r_t^* = (1 + x_t)(1 + r_t)^{1/(\varepsilon + 1)} - (1 + x_t + d_t)$$
. (5)

The foregoing analysis and equation (5) assume that the other cap authority x_t may be observed separately from the RRA and, in particular, is free of the induced density authority d_t . In practice, the other cap authority x_t cannot be observed totally independent of the RRA, since any RRA-induced volume loss is subsumed in the computation of the density authority. That is, we observe:

$$x_t^{obs} = CPI_t + DA_t^{obs} + PBA_t + Other_t$$
,

which is the sum of the CPI, observed density, performance-based, and other (if any) rate authority components. The observed density authority includes the density authority that would be obtained from any factors driving changes in density not related to the RRA, plus the induced density authority produced by the RRA (d_t):

$$DA_t^{obs} = DA_t^{NoRRA} + d_t$$
.

Thus, in terms of the notation above, all price authority x_t includes the density effect produced by the RRA (d_t):

$$x_t^{obs} = x_t + d_t .$$

Consequently, the rate authority from equation (5) would be implemented as

Formula IV-5a:
$$r_t^* = (1 + x_t^{obs} - d_t)(1 + r_t)^{1/(\varepsilon+1)} - (1 + x_t^{obs})$$
, (6)

where r_t is calculated according to the Commission's Formula IV-1, and d_t is defined in equation (2) above, or equivalently by:

Formula IV-6:
$$d_t = -(IC/TC)_t * (((1 + x_t^{obs} + r_{t-1}^*)/(1 + x_t^{obs} - d_t))^{\varepsilon} - 1)$$
. (2')

In this option, the required remittance continues to be computed using Formula IV-3 as proposed by the Commission.² It is only under the cap authority allowed by Formula IV-5a (in conjunction with the density adjustment via Formula IV-6) that the actual revenue generated by the RRA is sufficient to fund Formula IV-3's required remittance.

Note that even if the demand elasticity is zero, r_t^* does not reduce exactly to r_t , but rather to

$$r_t^* = r_t + r_t x_t - d_t = r_t + r_t x_t$$
 (if $\varepsilon = 0$).

The term $r_t x_t$ (which is small relative to r_t) arises because the components of the cap authority are additive. The second equality results from equation (2), where d_t is zero when demand is perfectly inelastic, as there is no volume loss due to the application of the RRA in this case. However, in the more general case, it still necessary to compensate for the interaction between the retirement and density-based authority terms to equate the remittance requirement and the generated revenue.

II. Option 2: Add a formula for the remittance obligation (r^{**}) funded by the Commission's proposed retirement rate authority on an after-rates basis

Conversely, if r_t is the given RRA, Formula IV-3 from the Commission's proposal needs to be modified to recognize that the amount generated by an r_t percent increase results in less revenue due to some volume loss. Equation (1) may also be used to solve for the revenue accumulation term, here labeled as r^{**}_t , to reflect the actual market dominant revenue yielded on an after-rates basis by r_t :

$$\left(\frac{r_t^{**}}{1 + r_t^{**}}\right) MDR(r_t)_{t,AR} = MDR(r_t)_{t,AR} - MDR_{t,BR}$$

or

$$\left(\frac{r_t^{**}}{1+r_t^{**}}\right) = 1 - MDR_{t,BR}/MDR(r_t)_{t,AR}.$$
 (1")

As with r_t^* , the revenue yield r_t^* is a function of the ratio of the demand elasticity and the cap authority available with and without the retirement authority:

$$r_t^{**} = (1 - ((1 + x_t)/(1 + x_t + d_t + r_t))^{\varepsilon + 1})/((1 + x_t)/(1 + x_t + d_t + r_t))^{\varepsilon + 1}.$$
 (7)

² In the event that the density authority were not present in the final rule, d_t would be zero and Formula IV-6 would be unnecessary.

Similar to equation (5), above, equation (7) assumes that the other cap authority x_t may be observed independent of the RRA. Since, as in Formula IV-5, x_t is observed inclusive of the induced density authority d_t , the revenue yield adjustment may be implemented equivalently as

Formula IV-5b: $r_t^{**} = (1 - \phi_t)/\phi_t$, (8)

where

$$\phi_t = ((1 + x_t^{obs} - d_t)/(1 + x_t^{obs} + r_t))^{\varepsilon+1}.$$

Note that $x_t^{obs} - d_t$ is the cap authority excluding the RRA – both direct and induced effects – and $x_t^{obs} + r_t$ is the cap authority including the RRA. The term d_t is computed using Formula IV-6, as with Option 1. If this option were chosen, the revenue yield r^{**}_t from Formula IV-5b would be substituted for r_t in the computation of revenue generated by the RRA, and hence the remittance required, under the Commission's Formula IV-3:

Formula IV-3b:
$$MDR_T(1 - (\prod_{t=T-N}^T (1 + p_t r_t^{**}))^{-1}).$$
 (9)

The terms MDR_T and p_t are defined in Order 5337 at page 98. Again, it should be noted that the revenue yielded by r_t (that is, r^{**}_t) is less than that yielded by $r^*_{t,t}$, proposed as "Option 1" in the preceding section. Thus, this alternative approach would fund a smaller portion of the relevant payments than would the first approach above.

III. Scenario Analysis and Examples

The accompanying Excel workbook demonstrates the over-remittance problem caused by the Commission's proposed formula and the effects of the two solutions described above. The scenarios use the starting revenue amounts and the series of amortization payments from Table IV-5 of Order 5337. Order No. 5337 at 100. Consequently, the initial value of r_t (0.861 percent) is identical to the example given in Table IV-6. *Id.* In addition, the scenarios employ a demand elasticity of -0.402, which is the volume-weighted average of the FY2019 market-dominant demand elasticities filed with the Commission on January 21, 2020. See tab "Elasticity". Subsequent values of revenues and r_t differ because the scenario analysis computes changes in market-dominant and total revenues on the basis of certain stylized assumptions:

- the application of specified amounts of other rate authority, representing hypothetical amounts of CPI, density-based, and performance-based authority (excluding the RRA) available to the Postal Service;³
- that market-dominant volume will decline exogenously at specified rates; and
- that competitive-product revenue will increase at specified rates.

³ The scenarios exclude additional rate authority for non-compensatory classes, for the sake of simplicity.

In the scenario analysis, the rate increases are assumed to be in effect for full fiscal years, so that the proration terms can be ignored for simplicity; as a result, the compounded previously authorized rate authority (PARA) and the percent of market-dominant revenue from Formula IV-3 are identical.

A. Scenario 1: Baseline Scenario per Order No. 5337, with RRA Only

In Scenario 1 (tab "RRA & Remittance 1"), the computation of after-rates revenue from the RRA omits the interaction between the RRA and the density-based authority. The average RRA authority, 0.87 percent (cell M25), is slightly lower than the 0.94 percent average in the Commission's Table IV-6 example, mainly due to the r_t rate authority being calculated on a higher total revenue base.

When fully phased in, the RRA yields \$1,266 million on an after-rates basis (cell Q24), but the mandated remittance is 4.26 percent of the resulting market-dominant revenue, or \$2,182 million (cells N24 and S24, respectively). In other words, after-rates revenue from the RRA funds only 58 percent of the required remittance (column W).

This degree of underfunding is consistent across all years of the phase-in period. The shortfall would be expected to persist insofar as the Commission's formula provides no additional RRA after year 5. Additionally, compounding additional years' rate authority in Formula IV-3 would simply serve to further increase the required remittance in subsequent periods.

Note that the amount of the shortfall is highly sensitive to the demand elasticity. (Indeed, expressed as a percentage, it closely approximates the inverse of the elasticity ratio.) This is because the after-rates revenue yielded by the RRA fluctuates with the demand elasticity, while the required remittance remains constant.

B. Scenario 2: Baseline Scenario with RRA-Induced Density Rate Authority

Scenario 2 (tab "RRA & Remittance 2") accounts for the interaction of the density and retirement rate authorities. The density authority interaction provides additional rate authority d_t due to price increases under the RRA reducing volumes entering the density calculation, with a lag, compared to a no-RRA baseline. Since the other cap authority excludes RRA impacts, by assumption, the induced density authority is calculated using equation (2) above, and is added to reflect the effect of the density-RRA interaction. The induced density authority ranges from 0.15 to 0.20 percentage points of additional cap authority in years 2-5 (column E).

Including the RRA and the induced density-based authority, the total additional cap space averages 1.01 percentage points for the five years (cell Y25). The additional cap authority increases revenue by \$197 million, compared with Scenario 1, when the RRA

is fully phased in (comparing cell Q24 in each tab), and the mandated remittance (column S) is little changed from Scenario 1.

Accounting for the interaction between the RRA and the density-based rate authority improves the remittance funding ratio over the period, but a substantial gap between the required remittance and the revenue generated by the RRA remains. The RRA still funds only 58 percent of the required remittance in the first year, and rises only to 67 percent when fully phased in, with an ultimate shortfall of \$724 million (columns W and U, respectively).

As in Scenario 1, the shortfall is highly sensitive to demand elasticity, as neither the rate authority nor the required remittance has yet been adjusted to account for after-rates effects.

C. Scenario 3: Adjusted RRA (r_t) per proposed Option 1

Scenario 3 (tab "RRA & Remittance 3") adjusts the authorized RRA such that the RRA produces the revenue yield given by r_t on an after-rates, rather than before-rates, basis. The adjusted rate authority r_t^* is given by equation (5) above. The computations of r_t and the accumulation of revenue in Formula IV-3 are otherwise unchanged from the Commission's proposal.

With the compensation for the demand elasticity, the adjusted RRA averages 1.51 percent, including induced density-based authority (cell Z25). This amount of rate authority completely funds the mandated remittance in each year, with the after-rates revenue and remittance requirement both \$2,202 million when the RRA is fully phased in (columns R and T, respectively).

It can be seen from altering the model parameters that the balance between the afterrates revenue from the RRA and the required remittance is not affected by the specific values of any of the scenario inputs, including the elasticity, the assumed rate of market-dominant volume loss, the CPI inflation rate, the rate of competitive revenue growth, or the institutional cost share.

⁴ For Scenario 3, equation (5) is used rather than equation (6) because the other cap authority values are posited excluding the induced density rate authority, which is added to obtain the total cap authority available under the RRA. As noted above, in practice, the RRA-induced density rate authority would not be observed separately from the overall density rate authority calculation, in which case equation (6) would define the cap authority.

D. Scenario 4: Adjusted Remittance Requirement (r^{**}_t) per proposed Option 2

Scenario 4 (tab "RRA & Remittance 4") shows the alternative of adjusting the remittance requirement rather than the RRA to balance the remittance requirement with after-rates revenue.

In Scenario 4, the average additional cap space from the RRA – including density effects – is the same 1.01 percentage points as in Scenario 2 (cell AA25). However, the required remittance formula IV-3 accumulates the after-rates revenue yields r^{**}_t based on equation (7), rather than the before-rates r_t values.⁵ Accordingly, at full phase-in, the required remittance is 2.84 percent of market-dominant revenue (cell Q24), rather than 4.25 percent in Scenario 2 (cell N24 in tab "RRA & Remittance 2").

Similar to Scenario 3, the after-rates revenue and required remittance are in balance each year (columns S and U, respectively), in this case due to the corrected compounding of the after rates yields from the RRA in the remittance formula. However, the remittance funded by the RRA is \$1,463 million (cell U24), approximately two-thirds of the \$2,202 million remittance funded under the rate authority adjustment in Scenario 3 and 45.7 percent of the total Year 5 amortization requirement of \$3,200 million in the Commission's Table IV-5. See Order No. 5337 at 100.

As in Scenario 3, the balance between the RRA revenue and remittance is robust to alternative values of the model inputs.

⁵ Similar to Scenario 3, as discussed in footnote 4 above, equation (7) is used rather than equation (8) because the other cap authority values are posited excluding the induced density rate authority for the purposes of the scenario.